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THE ROLE OF FAMILIES IN VALUING RISKS TO LIFE

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I. INTRODUCTION

In recent years the practice of imputing value to life based on lifetime earnings has been replaced by a life cycle consumption approach (Usher, 1973; Conley, 1976; Arthur, 1981, Shepard and Zeckhauser, 1982, 1984) which recognizes that it is not simply the present value of consumption which affects utility but how it is spread over time. According to this approach the amount which an individual should be willing to pay for a change in his current probability of death is the amount necessary to keep his expected utility of lifetime consumption constant.

Although this approach is an improvement over the human capital method, it has thus far ignored an important feature of reality--the fact that persons have dependents whom they care about--which significantly affects an individual's willingness to pay (WTP) to reduce his own risk of **death**.¹ The purpose of this paper is to examine WTP for a person who has "loved dependents" for at least part of his life and to contrast his WTP with that of a single individual.

To understand why altruism may significantly alter WTP, one can separate an individual's WTP into two parts--his "selfish" WTP, which reflects the utility he receives from his own consumption, and his WTP on behalf of his loved ones, which is a function of their expected utility. Holding the individual's income constant, his own consumption and, hence, his selfish WTP are almost surely lower if he has dependents than if he does not. This suggests that the WTP of a person without dependents should exceed the WTP of a person with dependents.

On the other hand, the person with dependents cares about his family, and will pay an amount over and above his selfish WTP provided that his family's consumption is higher when he is alive than when he is dead. Whether or not this is the case depends on capital market opportunities. When actuarially fair life insurance is available the altruistic individual purchases life insurance to the point where each dependent's consumption is the same whether he lives or

dies. Once all dependents have been born, therefore, WTP due to family motives is zero and altruism unambiguously reduces WTP.

The situation is somewhat different when life insurance is not available and borrowing opportunities are limited. In this case a person's dependents are likely to be financially better off when he is alive, at least at the beginning of his life, and the person's WTP on behalf of his dependents should thus be positive. The net effect of dependents on WTP is therefore ambiguous when a person is young.

The situation changes, however, as the individual builds up wealth to finance post-retirement consumption. With sufficiently large wealth a person's dependents are financially better off without him since, when he dies, they receive the money that would have financed his consumption. Once this happens WTP due to family motives is negative and dependents unambiguously reduce WTP.

To illustrate the magnitude of this effect, WTP is computed for an altruistic person and for a single individual assuming that both have isoelastic utility functions. The quantitative effects of altruism are striking. With perfect annuities markets, peak WTP is almost twice as large for a man without dependents as it is for a man with a wife and two children. When life insurance is unavailable and borrowing is not allowed, the maximum reduction in WTP is even greater. This suggests that treating all persons as single individuals when computing their WTP may seriously overstate the value of life.

To establish these results we examine the lifetime utility maximization problem of a person with dependents (hereafter termed the "head of household") and derive his WTP for a change in his conditional probability of death. This is contrasted with a single individual's WTP under alternative capital market assumptions. Analytical results appear in section II and numerical results in section III. Section IV concludes.

II. WTP FOR CHANGES IN THE LIFE EXPECTANCY OF A HEAD OF HOUSEHOLD

A. The Utility Maximization Problem

To calculate WTP it is necessary to solve the utility maximization problem for the head of household. The problem is simplified if we assume that the only uncertainty facing the head is his date of death. The age at which he marries and the size of his family in year t , m_t , are assumed known with certainty at the first planning date.² To facilitate comparisons with a single individual, we assume the household head is the only earner and that his family receives his labor income, y_t , only if he is alive.

Since the head of household is altruistic he maximizes the sum of the expected utilities of all family members. Let τ denote the first year in which planning occurs or, equivalently, the head's age at that time, and let k be the current year, $k \geq \tau$.³ If the head dies at the end of year t the family's utility, measured from k , is given by

$$U_t = \sum_{i=k}^t \alpha^{i-k} m_i u(c_i/m_i) + \alpha^{t-k} V_t(S_t), \quad (1)$$

$$u' > 0, \quad u'' < 0, \quad u''(0) = \infty, \quad v' \geq 0, \quad v'' \leq 0,$$

where $u(\cdot)$ is the period utility function for each individual in the family, α is a common subjective discount factor, $0 < \alpha \leq 1$, and $V_t(S_t)$ is the utility, discounted to year t , which the surviving members of the family receive from consuming estate S_t .

Assuming that survivors can lend at rate ρ , $V_t(S_t)$ is formally expressed as

$$V_t(S_t) = \max_{\{\tilde{c}_i\}} \sum_{i=t+1}^{T'} \alpha^{i-t} n_{i,t} u(\tilde{c}_{i,t}/n_{i,t}) \quad (2)$$

$$\text{s.t.} \quad \sum_{i=t+1}^{T'} r^{i-t} \tilde{c}_{i,t} = S_t, \quad r \equiv (1+\rho)^{-1},$$

where $n_{i,t}$ denotes the number of survivors in year i if the head dies at age t and $\tilde{c}_{i,t}$ their total **consumption**.⁴ T' is the date at which the last family member dies.

Since the head's date of death is uncertain (1) must be weighted by the probability that the head dies at age t . Let $p_{t,k}$ denote the probability that the head dies at age t (just before his $t+1^{st}$ birthday) given that he is alive at age k , and let T be the oldest age to which he can live. Assuming

$$p_{t,k} \geq 0, \quad t = k, \dots, T \quad \text{and} \quad \sum_{t=k}^T p_{t,k} = 1 \quad (3)$$

so that $\{p_{t,k}\}$ constitute a known probability distribution over length of life, the probability that the head survives at least to the beginning of year t , given that he is alive in year k , can be written

$$q_{t,k} \approx \sum_{i=t}^T p_{i,k} \quad (4)$$

The head's lifetime expected utility, evaluated at k , may now be written

$$J_k = \sum_{t=k}^T p_{t,k} U_t = \sum_{t=k}^T q_{t,k} \alpha^{t-k} m_t u(c_t/m_t) + p_{t,k} \alpha^{t-k} V_t(S_t). \quad (5)$$

Maximum expected utility and, hence, WTP at age k for a change in $\{p_{t,k}\}$, depend crucially on capital market opportunities. We consider two extreme cases. The first, and probably more realistic, is that the head does not have access to actuarially fair life insurance and annuities, but can borrow and lend at rate ρ . In this case an amount s_t ,

$$s_t = y_t - c_t,$$

is invested each period, yielding an estate at the end of year t of

$$S_t = S_{k-1}r^{k-t-1} + \sum_{i=k}^t r^{i-t}s_i, \quad t = k, \dots, T, \quad (7)$$

where $r^{-1}s_{k-1}$ represents the bequeathable wealth with which the head begins year k . We also assume that the head can never have negative net worth, i.e.,

$$S_t \geq 0 \quad \text{all } t, \quad (8)$$

to guarantee that he does not die insolvent. With no insurance markets the utility maximization problem is to select the consumption stream c_k ,

c_{k+1}, \dots, c_T which maximizes (5) subject to (6)-(8).

Alternatively, the head may be allowed to purchase actuarially fair life insurance and annuities, and to borrow via life-insured loans. To prevent the head from borrowing an infinite amount via such loans we require that the present value of expected consumption and investment in estate equals the head's expected lifetime earnings plus annuities held at the end of year k ,⁵

$$\sum_{t=k+1}^T r^{t-k} q_{t,k}(c_t + s_t) = A_k + \sum_{t=k+1}^T r^{t-k} q_{t,k}y_t. \quad (9)$$

Expected utility in this case is maximized subject to (7)-(9).

B. Evaluation of Willingness to Pay

Consider now a government program which reduces the head's risk of death at age j . This program affects the head's conditional probability of survival at age j , $q_{j+1,k}/q_{j,k}$, or equivalently, his conditional probability of death, D_j , where

$$1 - D_j \equiv q_{j+1,k}/q_{j,k}, \quad j = k, \dots, T. \quad 6$$

What should the head be willing to pay for a marginal reduction in D_j ? The wealth which can be taken away from the head and keep his utility constant is given by

$$WTP_{j,k} = - \frac{dJ_k/dD_j}{dJ_k/dW_k}, \quad k, j = \tau, \dots, T, \quad j \geq k \quad (11)$$

in which $WTP_{j,k}$ denotes WTP in year k for a change in the head's conditional probability of death in year j , and W_k denotes assets at the start of year k .⁷

1. WTP with Perfect Annuities Markets

To evaluate (11) in the case of perfect annuities markets one can apply the Envelope Theorem to the Lagrangian function

$$L_k = J_k + \lambda_k [A_k + \sum_{t=k+1}^T r^{t-k} q_{t,k} (y_t - c_t - s_t)] \quad (12)$$

to obtain

$$\begin{aligned} HWTP_{j,k} = & \lambda_k^{-1} \{ (1-D_j)^{-1} [\sum_{t=j+1}^T q_{t,k} \alpha^{t-k} m_t u(c_t/m_t) + p_{t,k} \alpha^{t-k} v_t(s_t)] - q_{j,k} \alpha^{j-k} v_j(s_j) \} \\ & + (1-D_j)^{-1} [\sum_{t=j+1}^T r^{t-k} q_{t,k} (y_t - c_t - s_t)], \end{aligned} \quad (13)$$

where H has been added to emphasize that this is the head of household's WTP. The first line of (13) says that for a small reduction in D_j the head will forfeit an amount equal to the present value of the change in his expected utility from period $j + 1$ onward. This amount must, however, be adjusted by the terms on the second line, which measure the effect of D_j on the budget constraint. A reduction in D_j makes the head wealthier by increasing the present value of his expected lifetime earnings from age $j + 1$ onward, and this increases

WTP. An increase in survival probabilities, however, decreases the consumption and investment in estate which the head can afford in years $j + 1, \dots, T$, and the head's WTP is reduced by these amounts.*

More insight into the effect of family motives on WTP can be gained by rewriting (13) slightly and comparing it to the corresponding expression for a single individual. The single person's WTP (SWTP) is obtained by setting $s_t = V_t(S_t) = 0$ and $m_t = 1$ in (13),

$$\begin{aligned} \text{SWTP}_{j,k} = & \lambda_k^{-1} \{ (1-D_j)^{-1} \sum_{t=j+1}^T q_{t,k} \alpha^{t-k} u(c_t) \} \\ & + (1-D_j)^{-1} \sum_{t=j+1}^T r^{t-k} q_{t,k} (y_t - c_t). \end{aligned} \quad (14)$$

The head's WTP can be rewritten as

$$\begin{aligned} \text{HWTP}_{j,k} = & \lambda_k^{-1} \{ (1-D_j)^{-1} \sum_{t=j+1}^T q_{t,k} \alpha^{t-k} u(c_t/m_t) \} + (1-D_j)^{-1} \sum_{t=j+1}^T r^{t-k} q_{t,k} [y_t - (c_t/m_t)] \\ & + \lambda_k^{-1} \frac{\partial}{\partial D_j} \{ \sum_{t=j}^{T'} q_{t,k} \alpha^{t-k} (m_t - 1) u(c_t/m_t) + \sum_{t=j}^{T'} p_{t,k} \alpha^{t-k} \sum_{i=t+1}^{T'} \alpha^{i-t} n_{i,t} u(\tilde{c}_{i,t}/n_{i,t}) \} \\ & - \frac{\partial}{\partial D_j} \{ \sum_{t=j}^{T'} r^{t-k} q_{t,k} [s_t + (m_t - 1)(c_t/m_t)] \} \end{aligned} \quad (15)$$

where the first line represents what the head would be willing to pay if he had no dependents (his "selfish" WTP) and the second and third lines represent the head's WTP on behalf of his family.

Notice that the portion of the head's WTP attributable to selfish motives is identical in form to the single individual's WTP. Since the consumption of the head, c_t/m_t , is uniformly below the consumption of a single person with equivalent earnings (see section A4 of the Appendix), dependents decrease an individual's WTP based on his own consumption.

The total effect of dependents, however, depends on the head's WTP on behalf of his family. Intuitively, this should be zero if dependents' consumption is unaffected by the head's death and positive (negative) if dependents consume less (more) if the head dies than if he lives. With perfect annuities markets consumption per dependent is unaffected by the head's death since in year j the head purchases life insurance to the point where

$$c_t/m_t = \tilde{c}_{t,j}/n_{t,j} \quad \text{all } t \geq j. \quad (16)$$

As long as no additions to the family are planned to occur after year j , i.e.,

$$m_t - 1 = n_{t,j} \quad \text{all } t \geq j, \quad (17)$$

all dependents will have been provided for through the purchase of life insurance by the time D_j is altered. The head's WTP on behalf of his family will, therefore, be **zero**.¹⁰

If, however, further additions to the family are planned to occur after j , the head's WTP on behalf of his family will be positive. The reason is that life insurance, which can provide for dependents once they are born, cannot guarantee that the head will live long enough to have dependents. A positive family motive when the head is young thus reflects his desire to stay alive long enough to marry and raise a family.

The effect of dependents on WTP is thus ambiguous before the last child is born. Dependents reduce WTP by reducing the head's consumption; however, the anticipation of dependents yet unborn increases the head's WTP for survival. Once the last child has been born only the first effect operates and altruism unambiguously reduces WTP.

2. WTP with No Insurance Markets

How are these results altered when the head cannot purchase actuarially fair life insurance or annuities? When insurance is unavailable the head's WTP is given by (18),

$$\begin{aligned} \text{HWTP}_{j,k}^* &= [u'(c_k/m_k)]^{-1} (1-D_j)^{-1} \sum_{t=j+1}^T q_{t,k} \alpha^{t-k} u(c_t/m_t) \\ &+ [u'(c_k/m_k)]^{-1} \{ (1-D_j)^{-1} [\sum_{t=j+1}^T q_{t,k} \alpha^{t-k} (m_t-1) u(c_t/m_t) + p_{t,k} \alpha^{t-k} V_t(S_t)] - q_{j,k} \alpha^{j-k} V_j(S_j) \}. \end{aligned} \quad (18)$$

As in the perfect markets case (see equation (13)), WTP equals the change in expected lifetime utility, both from the head's consumption and from his family's, resulting from a change in D_j .¹¹ The second line of equation (13) is, however, absent here since, by (6), $y_t = c_t + s_t$ for all t . For a single individual WTP is given by (18) with $S_t = V_t(S_t) = 0$ and $m_t = 1$,

$$\text{SWTP}_{j,k}^* = (1-D_j)^{-1} [u'(c_k)]^{-1} \sum_{t=j+1}^T q_{t,k} \alpha^{t-k} u(c_t). \quad (19)$$

As in the perfect markets case the single individual's WTP is identical in form to the head's WTP based on his own consumption (the first line of (18)). Thus, provided that the single individual's consumption exceeds the head's, dependents must lower the head's "selfish" WTP.¹²

Consider now the head's WTP on behalf of his family. Before all dependents have been born the head's WTP due to family motives should be positive, reflecting the head's desire to live long enough to have a family. Unlike the perfect markets case, however, the head's WTP due to family motives may remain positive even after all dependents have arrived. The reason is that, without life insurance, accumulating an estate is so **costly**¹³ that a family without inherited wealth is sure to consume more if the head lives than if he

dies, at least when the head is young. At the beginning of the life cycle the head's WTP on behalf of his family should, therefore, be positive and the relationship between the head's WTP and the single individual's ambiguous.

Eventually, however, WTP due to family motives must become negative. As the head accumulates wealth to finance post-retirement consumption his family may be financially better off if he dies than if he lives. (If the head dies the family shares the wealth that would have financed his consumption.) This clearly must occur by age 65, since all consumption must be financed out of wealth by that age, but it can occur earlier. Once the second line of (18) is negative the head's WTP must fall short of the single individual's, provided that the single person's consumption exceeds the head's.

The foregoing results suggest that dependents may increase WTP at the beginning of the life cycle but should decrease it later on. The timing of these effects is, however, less predictable than in the perfect markets case, and their magnitude is unknown in either case. To see more precisely how dependents affect WTP we must make specific assumptions about preferences and about the parameters of the life cycle model.

III. NUMERICAL RESULTS

A. Choice Of Parameters

To gain further insights into the life cycle behavior of WTP we turn to numerical solutions computed using the isoelastic utility function

$$u(c) = c^{\beta} / \beta, \quad 1 > \beta > 0. \quad (20)$$

β is constrained to be positive to insure that utility of consumption exceeds utility of death, which is implicitly zero for the selfish head of household and the single individual. The precise values of β chosen are 0.4, 0.2 and 0.1, which are consistent with empirical estimates of WTP based on labor market data and safety decisions.¹⁴ The values chosen for the discount factor

$(\alpha = (1.03)^{-1}, (1.05)^{-1}, (1.07)^{-1})$ allow the rate of time preference to be less than, equal to, or greater than the rate of interest, which is 5 percent.¹⁵

In all simulations the head of household is a white male with 1-3 years of college education. yearly earnings for men with this education level {U.S. Department of Commerce) were multiplied by employment rates, assumed zero before age 18 and after age 64, to yield effective earnings. Since consumption paths in the absence of insurance markets may be sensitive to the earnings profile, earnings were smoothed by regressing the logarithm of earnings on age and $(age)^2$,

$$\ln y_t = 7.1575 + 0.1378t - 0.001469t^2, \quad R^2 = 0.884. \quad (21)$$

Values predicted by (21) were used as $\{y_t\}$.

Mortality rates for white males between ages .18 and 100 (τ and T, respectively) were obtained from the U.S. Department of Health and Human Services. Together with $\{y_t\}$ they imply that expected lifetime earnings, discounted to age 18, are approximately \$400,000 1981 dollars.

To determine family size it was assumed, based on national demographic data, that the head at age 23 marries a 21-year-old woman, who bears him children when he is 25 and 28. In computing $\{m_t\}$ each adult 18 and older received a weight of 1.0. Each child received a weight of 0.3 through age 13 and of 0.62 between 14 and 17 (Dolde, 1978). Children were assumed to leave the household at age 22, and the wife assumed to die at age 78, implying $T' = 80$.

Since computation of WTP requires optimal consumption and estate streams, the head's lifetime consumption problem was solved beginning at age 18 assuming that inherited wealth at that time was zero.¹⁶ The resulting consumption and estate streams were used to evaluate WTP for a change in the current probability of death ($WTP_{j,j}$) at various ages.

B. The Effect Of Dependents On WTP

1. Perfect Annunities Markets

When life insurance is available we know that the single person's WTP exceeds the head's after the last child is born. What we do not know is how big this difference is or whether it holds at the beginning of the life cycle.

Figure 1 contrasts the head's WTP with that of a single individual when $\beta = 0.2$ and $\alpha = r = (1.05)^{-1}$. For the isoelastic utility function the ratio of the single person's consumption to the head's is constant for all t . When $\alpha = r$ the two consumption paths are themselves constant and independent of β . The single individual consumes \$20,133 per annum, while the head's desire to provide for his family reduces his consumption to \$9,097 per year.

This large discrepancy in consumption explains the huge gap between the head's WTP based on his own consumption and the single individual's WTP. This gap is widest in absolute terms at the beginning of the life cycle: however, family motives are also strong at the beginning of the life cycle. The result is that the head's willingness to pay exceeds the single individual's before age 23, although the percentage difference between the two is less than 15 percent.

After age 23, however, the single person's WTP exceeds the head's, with the difference between the two reaching a maximum at age 28 when the head's WTP on behalf of his family is zero. At this point the single person's WTP exceeds the head's by \$800,000 or 70 percent of the head's WTP.

As the head ages and his WTP falls the absolute difference between the single person's WTP and the head's also falls, although the percentage discrepancy increases. Note that once labor earnings cease WTP is proportional to the present value of expected consumption. Since the ratio of the single person's consumption to the head's is constant, the ratio of the single person's WTP to the head's is also constant and equal to the consumption ratio. This explains the constancy of the WTP ratios at the bottom of Table 1.

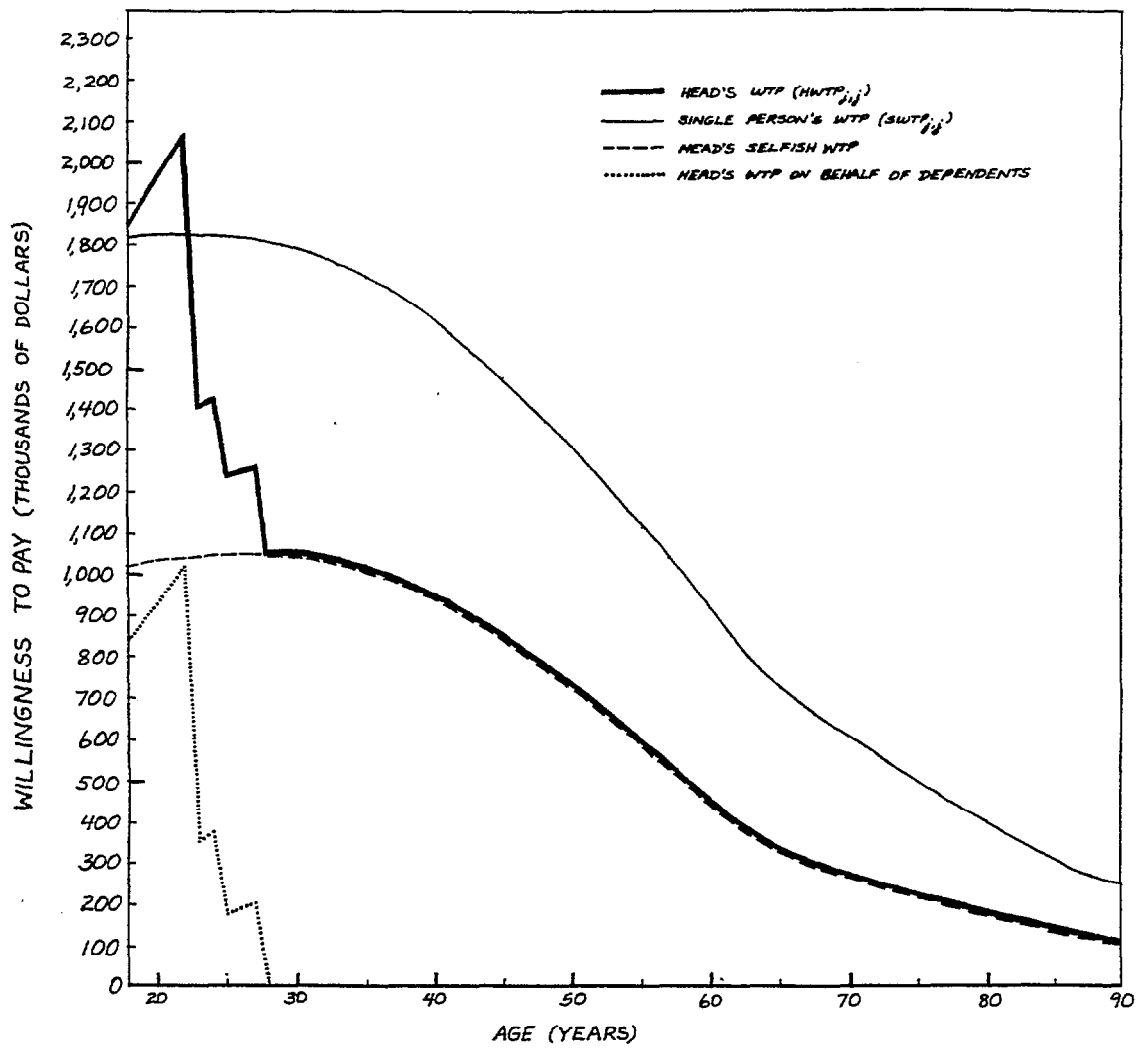


Figure 1. WILLINGNESS TO PAY, PERFECT ANNUITIES MARKETS
 $(\beta = 0.2, \alpha = r = (1.05)^{-1})$

TABLE 1
RATIO OF SINGLE PERSON'S WTP FOR CURRENT CHANGE
IN D_j TO HEAD'S WTP: PERFECT ANNUITIES MARKETS

Age (j)	$\beta = .10$			$\beta = .20$			$\beta = .40$		
				Subjective Discount Rate					
	.03	.05	.07	.03	.05	.07	.03	.05	.07
18	.981	.973	.966	.984	.976	.969	.990	.982	.975
20	.943	.920	.898	.952	.929	.907	.969	.947	.926
25	1.688	1.582	1.489	1.576	1.472	1.381	1.399	1.300	1.218
30	2.053	1.920	1.782	1.847	1.706	1.566	1.556	1.416	1.289
35	2.067	1.914	1.757	1.869	1.699	1.533	1.589	1.410	1.257
40	2.090	1.921	1.744	1.906	1.708	1.514	1.640	1.418	1.237
45	2.123	1.941	1.745	1.958	1.734	1.512	1.712	1.443	1.229
50	2.164	1.975	1.765	2.027	1.784	1.533	1.812	1.493	1.237
55	2.215	2.029	1.812	2.115	1.865	1.589	1.949	1.585	1.271
60	2.276	2.110	1.912	2.228	2.004	1.730	2.145	1.780	1.389
65	2.337	2.213	2.082	2.351	2.213	2.065	2.389	2.213	2.017
70	2.337	2.213	2.082	2.351	2.213	2.065	2.389	2.213	2.017
75	2.337	2.213	2.082	2.351	2.213	2.065	2.389	2.213	2.017
80	2.337	2.213	2.082	2.351	2.213	2.065	2.389	2.213	2.017
85	2.337	2.213	2.082	2.351	2.213	2.065	2.389	2.213	2.017
90	2.337	2.213	2.082	2.351	2.213	2.065	2.389	2.213	2.017

Figure 1 suggests that using WTP for a single person to measure what a head of household would pay to reduce his risk of death significantly overstates the latter. Table 1 indicates that these results are strengthened for more conservative individuals, i.e., for persons who are more risk averse or have larger values of α . Since the discrepancy between the single person's WTP and the head's is largely due to differences in per capita consumption, and since consumption is valued more highly relative to earnings the smaller is β , reducing β increases the ratio of the single person's WTP to the head's. Higher values of α increase the difference between the two willingnesses to pay because they widen the difference between the head's consumption and that of the single person.

2. No Annuities Markets

The effect which dependents exert on WTP still holds when insurance markets do not exist. Figure 2 and Table 2 indicate that, except at the beginning of the life cycle, the single individual's WTP greatly overstates the WTP of a person with dependents. There are, however, differences in the magnitude and timing of this effect between the insurance and no-insurance cases.

With no annuities markets dependents reduce the head's consumption, but the ratio of the single person's consumption to the head's is not constant. The result is that the difference between the single individual's WTP and the head's WTP based on his own consumption varies greatly over the life cycle. In Figure 2 the difference between the two is over \$1 million between the ages of 30 and 50. This is a time of peak consumption for the single individual but not for the head, who is forced to postpone consumption to build up an estate. After age 60 the difference between the head's selfish WTP and the single person's falls sharply since the two persons' consumption streams approach one another after that **time**.¹⁷

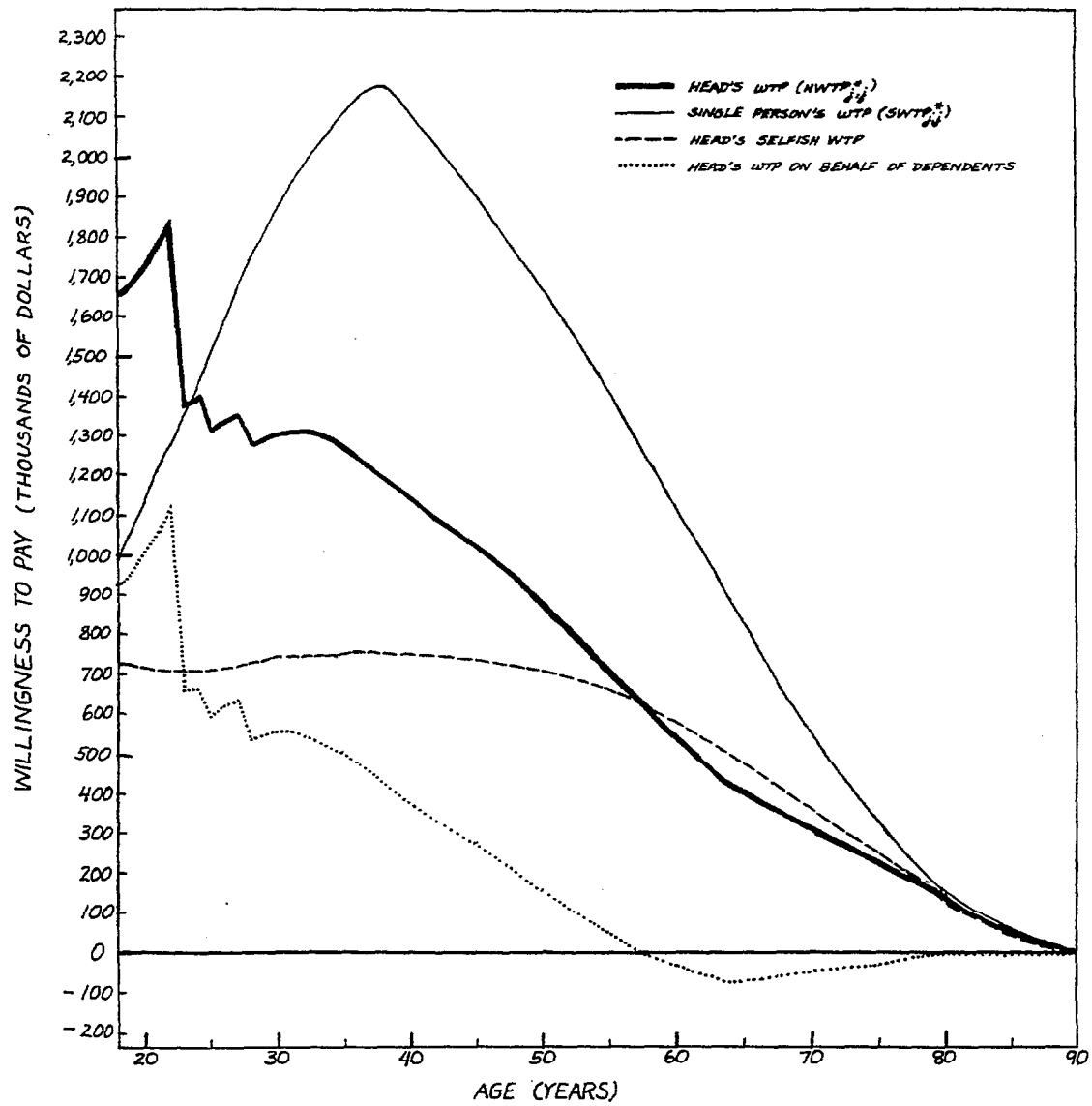


Figure 2. WILLINGNESS TO PAY, NO ANNUITIES MARKETS
 $(\beta = 0.2, \alpha = r = (1.05)^{-1})$

TABLE 2
RATIO OF SINGLE PERSON'S WTP FOR CURRENT CHANGE
IN D_j TO HEAD'S WTP: NO ANNUITIES MARKETS

Age (j)	$\beta = .10$			$\beta = .20$			$\beta = .40$		
				Subjective Discount Rate					
	.03	.05	.07	.03	.05	.07	.03	.05	.07
18	.737	.571	.512	.784	.599	.539	.897	.655	.622
20	.791	.627	.576	.830	.651	.601	.922	.707	.659
25	1.639	1.309	1.227	1.492	1.148	1.064	1.326	.994	.929
30	2.064	1.770	1.606	1.751	1.425	1.261	1.445	1.121	1.021
35	2.092	2.066	1.800	1.823	1.663	1.394	1.547	1.270	1.062
40	2.134	2.211	2.154	1.917	1.843	1.697	1.678	1.437	1.256
45	2.161	2.176	2.363	2.004	1.865	1.885	1.821	1.477	1.399
50	2.164	2.124	2.015	2.076	1.911	1.724	1.965	1.577	1.365
55	2.134	2.094	1.938	2.111	1.990	1.776	2.080	1.749	1.469
60	2.072	2.048	1.895	2.104	2.047	1.850	2.162	1.966	1.677
65	1.961	1.965	1.839	2.024	2.049	1.916	2.139	2.221	2.099
70	1.760	1.761	1.647	1.774	1.791	1.673	1.745	1.801	1.695
75	1.517	1.512	1.411	1.480	1.488	1.384	1.323	1.352	1.263
80	1.243	1.230	1.140	1.095	1.091	1.006	.737	.740	.681
85	1.243	1.230	1.140	1.095	1.091	1.006	.737	.740	.681
90	1.243	1.230	1.140	1.095	1.091	1.006	.737	.740	.681

The large discrepancy between the head's selfish WTP and the single person's overwhelms the head's WTP on behalf of his family, except at the beginning of the life cycle. As suggested in section II, family motives are most important when the head is young and his estate small. Before age 25 WTP on behalf of dependents is high enough that the head's WTP exceeds that of the single individual. The difference between the two, which is \$650,000 at age 18, is much greater than in the perfect markets case. After age 25, however, the head's WTP is below that of the single individual, with the latter exceeding the former by more than \$750,000 between the ages of 34 and 53. The largest discrepancy between the single individual's WTP and the head's occurs at age 38 when the difference between the two is \$1 million or 83 percent of the head's WTP. The difference between the single person's WTP and the head's declines with age but is still greater than in the perfect markets case at age 60.

The conclusion to be drawn from Figure 2 is that the difference between the single individual's WTP and the head's is larger than in the perfect market case during years of peak WTP. As in the case of perfect capital markets, the discrepancy between the two is more pronounced for smaller values of β (see Table 2). Changing α , however, has a smaller effect when annuities markets are absent than when they are present, especially for $\beta \leq 0.2$. This is intuitively reasonable. One way in which α affects WTP is by altering the time path of consumption; however, when annuities markets do not exist and borrowing is impossible, opportunities for rearranging consumption are limited.

C. The Effect Of Capital Markets On WTP

Section B implies that altruism has a large effect on WTP regardless of what one assumes about capital markets. In closing, it is interesting to note that capital market assumptions have a much smaller effect on the head's WTP. Figure 3 indicates that when $\alpha = r$ and $\beta = 0.2$ the maximum difference between the head's WTP with and without annuities markets is \$300,000 or 25 percent of the head's WTP. This occurs at age 35 when family motives cause WTP in the

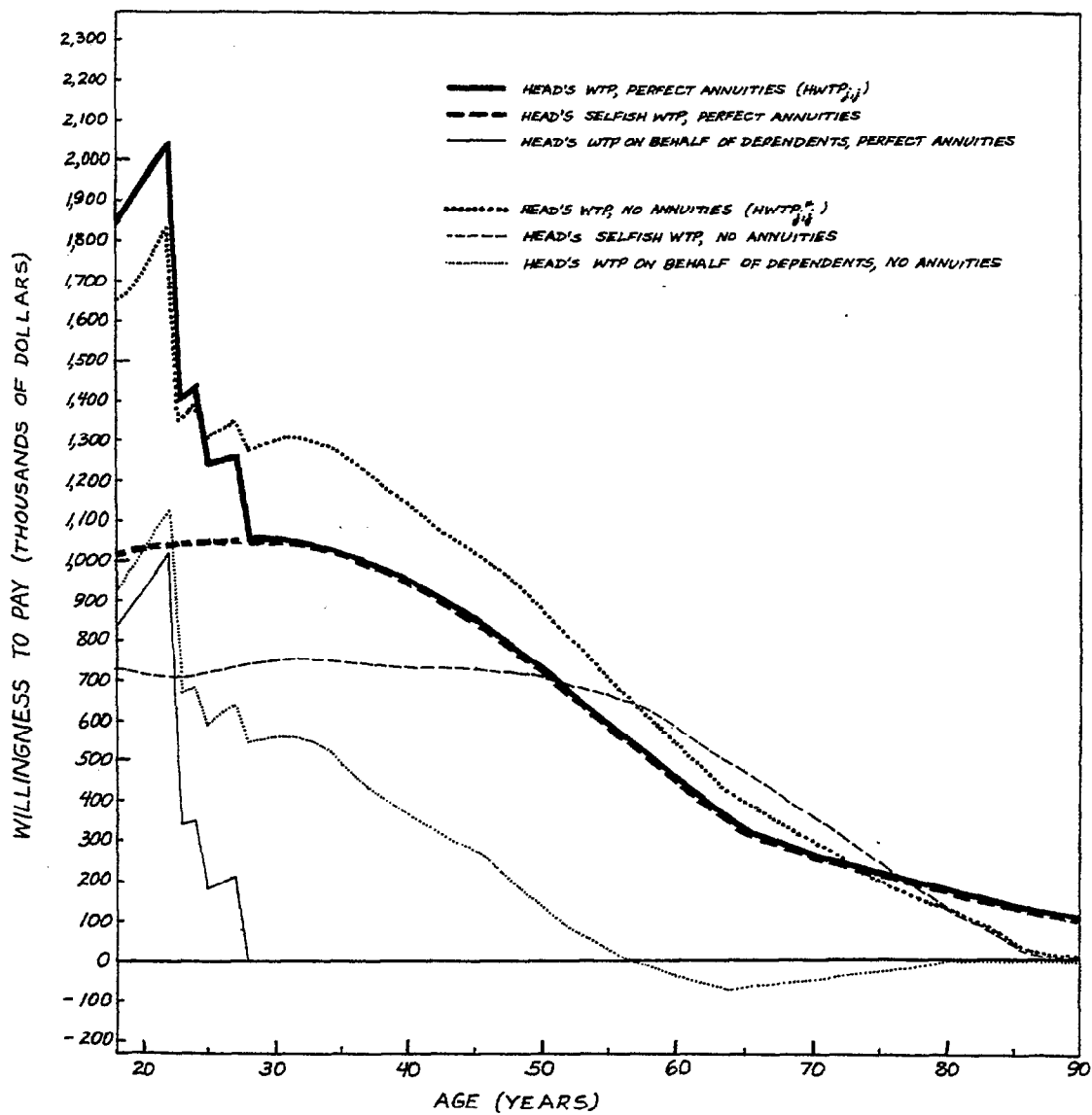


Figure 3. WILLINGNESS TO PAY WITH AND WITHOUT PERFECT ANNUITIES MARKETS ($\beta = 0.2$, $\alpha = r = (1.05)^{-1}$)

absence of annuities markets to exceed WTP in the perfect markets case. When the head's rate of discount exceeds the rate of interest (see Table 3) the maximum difference is similar in amount (\$331,000) but larger in percentage terms since persons with higher discount rates have lower willingnesses to pay. Conversely, when the discount rate is below the rate of interest WTP without annuities markets exceeds WTP with perfect annuities markets by at most \$160,000, or 13 percent of WTP.

The reason why the absence of annuities markets has such a small impact on the head's WTP is that it affects WTP in opposing ways. Until the head of household is in his late 50's lack of life insurance increases his WTP due to family motives compared to the head who can purchase actuarially fair insurance. After that time WTP due to family motives is zero or negative. At the same time the inability to purchase life insurance causes the head to postpone consumption so as to build up an estate. This reduces the head's "selfish" WTP early in life and increases it later in life, which offsets the first effect. The net result is that the head's WTP with perfect annuities markets is slightly greater than the WTP of the head without access to these markets early and late in life, but lower than the latter's WTP between the ages of 30 and 70.

IV. CONCLUSION

Once an economist has decided to impute a value to life using expected utility of consumption he faces the difficult task of determining the parameters which characterize preferences. The point made above is that once these parameters have been chosen and lifetime earnings determined the value imputed to life depends on what one assumes about family circumstances. For a head of household with a wife and two children using WTP formulas for a single individual may overstate the head's WTP by more than 80 percent during years of peak WTP.

TABLE 3
FOR CURRENT CHANGE IN D_j , RATIO OF HEAD'S WTP
(NO ANNUITIES MARKETS) TO HEAD'S WTP (PERFECT ANNUITIES MARKETS)

Age (j)	$\beta = .10$			$\beta = .20$			$\beta = .40$		
				Subjective Discount Rate					
	.03	.05	.07	.03	.05	.07	.03	.05	.07
18	.953	.864	.729	.968	.886	.742	.992	.925	.740
20	.952	.863	.727	.970	.885	.741	1.001	.927	.764
25	1.031	.981	.859	1.079	1.059	.933	1.099	1.122	.973
30	1.083	1.136	1.111	1.126	1.239	1.221	1.119	1.280	1.201
35	1.070	1.172	1.291	1.092	1.247	1.390	1.076	1.269	1.372
40	1.055	1.164	1.325	1.060	1.203	1.369	1.039	1.207	1.332
45	1.049	1.182	1.405	1.041	1.198	1.417	1.015	1.188	1.338
50	1.049	1.212	1.634	1.030	1.194	1.597	.999	1.153	1.504
55	1.050	1.226	1.707	1.028	1.182	1.603	.995	1.113	1.445
60	1.043	1.243	1.800	1.025	1.205	1.722	.995	1.135	1.560
65	1.009	1.235	1.920	1.000	1.240	2.068	.975	1.245	2.596
70	.923	1.131	1.760	.908	1.126	1.881	.867	1.110	2.322
75	.795	.977	1.524	.772	.961	1.610	.712	.916	1.926
80	.613	.759	1.191	.617	.775	1.308	.625	.814	1.732
85	.310	.383	.600	.282	.354	.596	.213	.276	.587
90	.098	.121	.189	.076	.095	.161	.036	.047	.099

The fact that the head's WTP is less than that of his single counterpart may seem surprising to persons who feel that a head of household has more "reason for living" than a person without dependents. One must, however, keep in mind what the life cycle consumption model can and cannot measure.

The life cycle model captures some of the desire to live longer through the intertemporal elasticity of substitution, equal to $(1-\beta)^{-1}$ for the isoelastic utility function. An individual who cares about length of life rather than total consumption views consumption when young as an imperfect substitute for consumption when old. The more imperfect this substitutability the lower is the elasticity of substitution and the higher is WTP.

The life cycle model does not, however, attach a value to length of life independently of amount consumed. As Bergstrom (1982) has pointed out, the intertemporal objective function, if derived from preferences on lotteries, should include a term which values survival independently of consumption. Whether this term should be higher for a head of household than for a single individual is not for an economist to say.

Some people may also believe that other persons' WTP to reduce A's risk of death should be higher if A has dependents than if he does not. This may well be true, but the only portion of dependents' WTP on the head's behalf which the above model does not capture is WTP arising from nonpecuniary motives. The financial loss which dependents may suffer if the head dies and any WTP arising therefrom are already included in the **model**.¹⁸

FOOTNOTES

1. A notable exception is Bergstrom (1982), who takes dependents into account when comparing WTP with the corresponding change in expected lifetime earnings.
2. If we were to allow the life spans of the spouse and children to be random variables, solving the utility maximization problem would require stochastic dynamic programming (for example, see Kotlikoff and Spivak, 1981). Even assuming an isoelastic utility function, the solution to this problem is intractable when there are labor earnings in addition to inherited wealth.
3. The solution to the head's utility maximization problem is consistent, so that plans made at τ for consumption and investment at ages k, \dots, T are unchanged if replanning occurs at k . The utility maximization problem is presented for $k \geq \tau$ to facilitate evaluating WTP at any age k .
4. To illustrate how $n_{i,t}$ varies with t , suppose that the head plans to marry at age 25. For $t < 25$, $n_{i,t}$ equals zero for all i and $V_t(S_t) \neq 0$. If $t \geq 25$, then $n_{i,t} \geq 1$ (strict inequality holds if one or more children have been born by t).
5. For details, see section A1 of the Appendix.
6. Since (10) implies that

$$q_{j,k} = \prod_{t=k}^{j-1} (1-D_t)$$

a change in D_j affects all subsequent survival probabilities $q_{j+1,k}, \dots, q_{j+1,T}$.

7. When there are no insurance markets $W_k = S_{k-1}r^{-1}$. In the perfect markets case $W_k = S_{k-1}r^{-1} + A_{k-1}(1+R_{k-1})$.
8. The budget constraints in section A insure that the head of household bears the consumption cost of any increases in life expectancy. Equation (13) therefore places no burden on future generations in the sense of Arthur (1981).
9. Equation (16) follows from the necessary conditions for the head's utility maximization problem ((A10) and (A11) in the Appendix) and a necessary condition implied by (2). Equations (A10) and (A11) imply that

$$V'_j(S_j) = (\alpha/r)^{t-j} u'(c_t/m_t), \quad t \geq j,$$

whereas (2) requires that

$$V'_j(S_j) = (\alpha/r)^{t-j} u'(\tilde{c}_t/n_{t,j}).$$

10. This result is proved in section A2 of the Appendix.
11. To see that (18) is identical in form to the first line of (13) note that in (13) $\lambda_k = u'(c_k/m_k)$ along an optimal path.
12. Although it is likely that the head's consumption is lower than the single individual's, especially at the beginning of life, this need not be true at all ages. If a single person with a high discount rate ($\alpha \leq r$) cannot purchase annuities he will try to consume most of his wealth while he is young. A head of household with the same survival probabilities and rate of discount has similar inclinations, but they conflict with his desire to build up an estate. If the latter is strong the head's consumption will peak late in life and may exceed that of the single individual in old age.
13. When life insurance is unavailable, accumulating one dollar of estate means foregoing one dollar of consumption. This may be contrasted with the cost of actuarially fair life insurance, which is

$$\phi_t = D_t(1+\rho)(1-D_t)^{-1}$$

per dollar at age t . For a white male aged 40, $\phi_t = .0027$.

14. Rosen (1985) computes an upper bound to β of 0.25-0.40 based on Ippolito and Ippolito's (1984) study of consumer reaction to the hazards of smoking and on wage differentials reported in Thaler and Rosen (1976).
15. WTP was also computed assuming $\rho = 0.03$ with the α/r ratios identical to those in Tables 1-3 below. Results were very close to those reported in the tables.
16. Solutions to the head's problem with and without annuities markets are outlined in section A3 of the Appendix.
17. The single person is reducing his consumption to reduce the size of unintended bequests, whereas the head is enjoying consumption which was postponed while he was building up an estate.
18. Another way of interpreting equations (13) and (18) is to say that they represent the sum of the head's selfish WTP and the WTPs of selfish family members for a change in the head's risk of death.

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APPENDIX

A1. In the perfect markets case, following Yaari (1965), we allow the head of household to purchase or sell both regular notes, which bear an interest rate p , and actuarial notes, which are repaid only if the head survives to the following year.¹ Since the latter are assumed actuarially fair, their interest rate, R_i , is implicitly defined for year i by

$$1 + R_i = (1+p)(1-D_i)^{-1}.$$

To prevent the individual from borrowing an infinite amount via life insured loans (issuing an infinite number of actuarial notes) we require that the stock of actuarial notes at the end of year T , A_T , be non-negative. Since $y_t - c_t - s_t$ denotes the amount invested in actuarial notes at the beginning of year t this implies

$$A_T = A_k \left[\prod_{j=k}^{T-1} (1+R_j) \right] + \sum_{j=k+1}^{T-1} (y_j - c_j - s_j) \left[\prod_{i=j}^{T-1} (1+R_i) \right] + y_T - c_T - s_T \geq 0. \quad (A1)$$

Since we have assumed $u' > 0$, (A1) will be binding. We can multiply (A1) by

$$\prod_{j=k}^{T-1} (1+R_j)^{-1} = q_{T,k} r^{T-k}$$

and regroup terms to yield

$$\sum_{t=k+1}^T r^{t-k} q_{t,k} (c_t + s_t) = A_k + \sum_{t=k+1}^T r^{t-k} q_{t,k} y_t. \quad (A2)$$

¹ Buying an actuarial note is equivalent to purchasing an annuity, whereas selling one constitutes borrowing via a life-insured loan. Financing the purchase of a dollar of regular notes with the sale of a dollar of actuarial notes is equivalent to purchasing a dollar of life insurance and paying a premium $\phi_i = R_i - p$.

A2. To see that equations (16) and (17) render the second line of (15) zero, note that their substitution reduces the term in braces to

$$\sum_{t=k}^{T'} q_{t,k} \alpha^{t-k} (m_t - 1) u(c_t / m_t) + \sum_{t=k}^{T'} p_{t,k} \alpha^{t-k} \sum_{i=t+1}^{T'} \alpha^{i-t} (m_i - 1) u(c_i / m_i). \quad (A3)$$

By changing the order of summation in the second line of (15) and rearranging terms the dependents' lifetime utility can be written

$$\begin{aligned} & q_{k,k} (m_k - 1) u(c_k / m_k) + \sum_{t=k+1}^{T'} [(m_t - 1) u(c_t / m_t) \alpha^{t-k} (\sum_{i=k}^{t-1} p_{i,k} + q_{t,k})] \\ &= \sum_{t=k}^{T'} (m_t - 1) u(c_t / m_t) \alpha^{t-k}, \end{aligned} \quad (A4)$$

where the equality follows from (3) and (4). Since lifetime utility is certain, its derivative with respect to D_j is zero.

To demonstrate that the third line of (15) is zero it suffices to show that (16) and (17) imply

$$s_t = -(m_t - 1) u(c_t / m_t), \quad \text{all } t \geq j. \quad (A5)$$

Substituting (16) and (17) into the budget constraint of (2) yields

$$\sum_{i=t+1}^{T'} r^{i-t} (m_i - 1) (c_i / m_i) = S_t. \quad (A6)$$

Multiplying (A6) for year $t-1$ by r^{-1} and subtracting from (A6) yields (A5).

A3. The solution to the head's maximization problem with perfect annuities markets is more transparent if the budget constraint (9) is rewritten. Repeated use of (A7) and (A8),

$$p_{t,k} = q_{t,k} - q_{t+1,k} \quad (A7)$$

$$s_t = s_t - r^{-1} s_{t-1}, \quad (A8)$$

which are derived from (4) and (7) respectively, permits the budget constraint to be written

$$w_k + \sum_{t=k}^T r^{t-k} [q_{t,k}(y_t - c_t) - p_{t,k} s_t] = 0. \quad (A9)$$

First-order conditions for maximization of (5) subject to (A9) require that (A10) and (A11) hold,

$$u'(c_t/m_t) = r\alpha^{-1} u'(c_{t-1}/m_{t-1}), \quad k \leq t \leq T \quad (A10)$$

$$u'(c_t/m_t) = v'_t(s_t), \quad k \leq t \leq T. \quad (A11)$$

Together, (A10) and (A11) permit c_t and s_t to be expressed as functions of c_k . Substitution into (A9) yields c_k .

In the case of the isoelastic utility function

$$u(c_t/m_t) = (c_t/m_t)^\beta \quad (A12)$$

and

$$v_t(s_t) = \alpha^{-t} r^{\beta t} \left[\sum_{i=t+1}^T \alpha^{i/(1-\beta)} r^{-\beta i/(1-\beta)} n_{i,t} \right]^{1-\beta} s_t^\beta \quad (A13)$$

$$= \alpha^{-t} r^{\beta t} \delta_t^{1-\beta} s_t^\beta.$$

Substitution of (A10)-(A13) into (A9) yields

$$c_k = \frac{w_k + \sum_{t=k}^T r^{t-k} q_{t,k} y_t}{\sum_{t=k}^T r^{t-k} \left(\frac{\alpha}{r}\right)^{(t-k)/(1-\beta)} \left[q_{t,k} \frac{m_t}{m_k} + p_{t,k} \delta_t^{1/(1-\beta)} m_t^{-1} \right]}. \quad (A14)$$

The solution to the single individual's problem is characterized by (A10) and (A14) with $m_k = m_t = 1$ and $\delta_t = 0$.

When insurance markets do not exist J_k is maximized subject to (6)-(8). A necessary and sufficient condition for (8) to be satisfied in years $T'+1, \dots, T$ is that $S_T = 0$. This implies (assuming $y_t = 0, t \geq T'$) that

$$\sum_{i=T'+1}^T r^{i-T'} c_i = S_{T'}.$$

Maximization of J_k subject to (6), (7), (A15) and $S_t \geq 0, k \leq t \leq T'$, requires that

$$u'(c_t/m_t) = (1-D_{t-1})^{-1} r \alpha^{-1} [u'(c_{t-1}/m_{t-1}) - D_{t-1} V'_{t-1}(S_{t-1})], \quad k \leq t \leq T, \quad (A16)$$

provided that $S_t > 0$. When $S_t = 0, c_t = y_t$.

When $S_t > 0$ for all $t < T$, (A16) together with $S_T = 0$ and w_k determine the optimal consumption **stream**.² Since a closed-form solution for $\{c_t\}$ is impossible the problem may be solved by guessing c_k , using (6) and (7) to determine S_k , and using (A16) to find c_{k+1} . In this manner the entire $\{c_t, S_t\}$ can be found. If $S_T = 0$ then c_k is correct. If S_T is positive (negative) then the initial guess should be raised (lowered) until $S_T = 0$.

² When $T' \geq T$ (A16) and the terminal condition $S_T = 0$ are replaced by $u'(c_T/m_T) = V_T'(S_T)$.

If the above procedure implies $S_t < 0$, which can occur only at the beginning of the horizon if $\{y_t\}$ is single-peaked, then one imposes the constraint $c_t = y_t$, $t = k, \dots, \hat{t}$, guesses $\hat{c}_{\hat{t}+1}$, and proceeds as above. Initially $\hat{t} = k$. \hat{t} is increased, one year at a time, until (8) is satisfied.

If (8) is never binding the solution to the single individual's problem is characterized by (A16), with $V_t' = 0$ and $m_t = 1$, and by $S_T = 0$. The latter implies that

$$W_k + \sum_{t=k}^T (y_t - c_t) r^{t-k} = 0. \quad (A17)$$

Since (A16) can be used to solve for c_t in terms of c_k , (A17) yields a closed-form solution for c_k .

If this approach results in $S_t < 0$, one must impose the constraint $c_t = y_t$, $t = k, \dots, \hat{t}$, in the manner described above and use (A16) and (A17) to determine $\hat{c}_{\hat{t}+1}$.

A4. We wish to demonstrate that the head's consumption is always less than that of a single person with identical earnings and initial wealth when there exist perfect annuities markets. Since the constraint (A2) applies to both persons, the assumption of identical incomes and initial wealth implies

$$\sum_{t=\tau}^T c_t^S q_{t,\tau} r^{t-\tau} = \sum_{t=\tau}^T [c_t^H/m_t + (m_t-1)(c_t^H/m_t) + s_t] q_{t,\tau} r^{t-\tau}, \quad (A18)$$

where superscripts S and H denote consumption for the single individual and household, respectively. Since it can be shown that

$$(m_t-1)c_t^H/m_t + s_t \geq 0, \quad \text{all } t, \quad (A19)$$

with strict inequality holding for at least one t , (A18) implies

$$\sum_{t=\tau}^T (c_t^S - c_t^H/m_t) q_{t,\tau} r^{t-\tau} > 0. \quad (\text{A20})$$

By (A20) there must be some age b at which $c_b^S > c_b^H/m_b$. (A10), however, implies that if the single person's consumption exceeds the head's in one year it must exceed it in all years.